Direct- and Quarter-Wave-Coupled Microwave Band-Pass Filters with Adjustable Transmission Characteristics and Fixed Center Frequencies*

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Summary—A theoretical study of the properties of "constant-phase" two-ports is presented which leads to a simple experimental synthesis technique for a broad class of microwave band-pass filters. "Constant-phase" two-ports consisting of modified step-twist-junctions are employed in experimental multiple cavity direct-coupled filter configurations which have the unique property of continually adjustable bandpass characteristics and fixed center frequency. These experimentally obtained characteristics are predicted from the theory.

ONE OF THE PRINCIPAL difficulties encountered in the design of microwave filters is the necessity of obtaining very precise physical dimensions, particularly those of the irises or junctions employed as reflecting elements. The electrical lengths of line between adjacent junctions can usually be adjusted conveniently with a tuning screw but changes in the physical dimensions of the junctions themselves usually involve machine shop work which is costly and time consuming. An extra degree of precision is required in general for direct-coupled filters[1] due to the difficulty of obtaining quantitative estimates of departures from design from an *over-all* filter characteristic, as opposed to obtaining specific bandwidths and resonant frequencies of a set of *individual* cavities in the more conventional quarter-wave coupled design.[2] (A rather complicated alignment and adjustment technique for direct coupled filters has been developed by Dishal[3] which moderates this problem to a degree.)

This paper is concerned with the properties of cascades of a particular type of junction labeled a "constant-phase" two-port. These cascades can provide both quarter-wave and direct-coupled microwave filter structures which to a large degree eliminate the problems discussed above. It has been shown[4] that two of these junctions can provide a single cavity filter of fixed center frequency and variable bandwidth. It will be shown here that quarter-wave coupled cascades of such cavities allow one to continuously vary the bandwidth of the individual cavities while maintaining quarter-wave coupling at the center frequency. It will further be shown that the quarter-wave coupling sections and their adjacent constant-phase two-ports can be replaced by a single constant-phase two-port yielding a direct coupled configuration, which allows a continuous variation of the coupling between adjacent cavities, while maintaining the correct pre-adjusted electrical spacing for a symmetrical response about a given frequency. (This spacing is identical, furthermore, for all cavities.) This permits a single filter to generate a tremendous variety of filter characteristics all centered at the same frequency. This also makes possible the direct synthesis of a given filter response curve without resort to an elaborate design technique, *i.e.*, it is a matter of a few minutes work with a swept-frequency oscillator to make electrical adjustments to obtain, say, a 3-cavity direct-coupled filter with 300 Mc bandwidth and 0.3-db ripple in the passband, as opposed to the usual procedure of first obtaining low frequency prototype parameters, then equivalent microwave structure parameters, machine shop execution of the design, and finally testing and tuning up the resulting piece of hardware. Furthermore, if the selectivity of the filter in actual use turns out to be inappropriate, it is again a matter of a few minutes work to change the bandwidth of the same filter structure to 200 Mc with 0.3-db ripple or some other ripple if desired. With usual techniques this would, of course, require a completely new filter. Thus these cascades allow the experimentor a tremendous flexibility in, and rapid attainment of, desired filter responses.

The "constant-phase" two-ports employed in the experimental cascades discussed are modified rectangular waveguide step-twist junctions.[4] The range of frequencies and bandwidths over which these junctions approximate the constant-phase assumption is discussed in the experimental section following a theoretical development of the properties of ideal constant-phase two-ports in cascade.

Theory

We will define a "constant-phase" two-port as one that is lossless, reciprocal, symmetrical, and has the property that its voltage reflection coefficient is represented by $R = |R| e^{j\theta}$ for all frequencies with $|R|$ and θ independent of frequency, and with $|R|$ in general variable from 0 to 1, but with θ a constant independent

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[1] S. B. Cohn, "Direct-coupled-resonator filters," Proc. IRE, vol. 45, pp. 187–196; February, 1957.

[2] W. W. Mumford, "Maximally-flat filters in waveguide," *Bell Sys. Tech. J.*, vol. 27, pp. 684–713; October, 1948.

[3] M. Dishal, "Alignment and adjustment of synchronously tuned multiple-resonant-circuit filters," Proc. IRE, vol. 39, pp. 1448–1455; Nov., 1951.

[4] B. C. DeLoach, Jr., "Step-twist junction waveguide filters," IRE Trans. on Microwave Theory and Techniques, vol. 9, pp. 130–135; March, 1961.

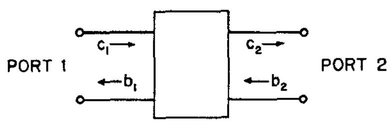

Fig. 1—Two-port representation.

of $|R|$. It can be shown that for such a two-port (see Fig. 1) the normalized voltage waves c_1 and c_2 travelling to the right at ports 1 and 2, respectively, and the normalized voltage waves b_1 and b_2 travelling to the left are related by[5]

$$\begin{bmatrix} c_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} c_2 \\ b_2 \end{bmatrix}$$

where the wave matrix $[A]$ is given by

$$[A] = \frac{1}{T}\begin{bmatrix} 1 & -R \\ R & -e^{2j\theta} \end{bmatrix}, \tag{1}$$

and T and R are the voltage transmission and reflection coefficients of the two-port. In order to cascade two such two-ports separated by a length of lossless transmission line, we need the $[A]$ matrix for such a length of line which is[5]

$$[A] = \begin{bmatrix} e^{j\beta l} & 0 \\ 0 & e^{-j\beta l} \end{bmatrix}. \tag{2}$$

Then, for two constant-phase two-ports, with the same θ but generally different $|R_i|$ and $|T_i|$, separated by such a length of line, we can write the wave matrix of the cascade as the matrix product of the wave matrices of the individual elements and obtain

$$A_{\text{total}} = \frac{e^{j\beta l}}{|T_1T_2|\, e^{j(\phi_1+\phi_2)}}\begin{bmatrix} 1 - |R_1R_2|\, e^{2j(\theta-\beta l)} & (-|R_2| + |R_1|\, e^{2j(\theta-\beta l)})e^{j\theta} \\ (|R_1| - |R_2|\, e^{2j(\theta-\beta l)})e^{j\theta} & -|R_1R_2|\, e^{2j\theta} + e^{2j(2\theta-\beta l)} \end{bmatrix} \tag{3}$$

where the subscript 1 refers to the two-port on the left, the subscript 2 to that on the right, with $|R_1|$, $|R_2|$, $|T_1|$, and $|T_2|$ the magnitudes of the reflection and transmission coefficients, and with ϕ_1 and ϕ_2 the phase angles of the transmission coefficients of the 1st and 2nd constant-phase two-ports, respectively. We can then write the complex voltage transmission and reflection coefficients of the cascade as

$$\begin{aligned} T_{\text{total}} &= \frac{1}{A_{11}} = \frac{|T_1T_2|\, e^{j(\phi_1+\phi_2-\beta l)}}{1 - |R_1R_2|\, e^{2j(\theta-\beta l)}} \\ R_{\text{total}(1)} &= \frac{A_{21}}{A_{11}} = \frac{(|R_1| - |R_2|\, e^{2j(\theta-\beta l)})e^{j\theta}}{1 - |R_1R_2|\, e^{2j(\theta-\beta l)}} \\ R_{\text{total}(2)} &= \frac{-A_{12}}{A_{11}} = \frac{(|R_2| - |R_1|\, e^{2j(\theta-\beta l)})e^{j\theta}}{1 - |R_1R_2|\, e^{2j(\theta-\beta l)}} \end{aligned} \tag{4}$$

[5] G. L. Ragan, "Microwave Transmission Circuits," McGraw-Hill Book Co., New York, N. Y., pp. 551–554; 1948.

where $R_{\text{total}(1)}$ is the reflection coefficient of the cascade from the end containing the two-port labeled 1, and $R_{\text{total}(2)}$ is that from the end containing the two-port labeled 2.

These coefficients are observed to have the remarkable property that their amplitudes which are symmetric about the same set of frequencies, obtained from $\theta-\beta l = K\pi/2$ with K any integer, for all values of $|R_1|$ and $|R_2|$. Similarly, it can be shown that any number of these two-ports, all with the same θ and all separated by identical lengths of transmission line (all sections, of course, having the same characteristic impedance), produces a cascade whose reflection and transmission characteristics are always symmetric in magnitude about this same set of frequencies. Note that this set of frequencies is not in general a set of passband frequencies. For instance, the two-element cascade represented by (4) in general has a passband only when $|R_1| = |R_2|$ and $\theta-\beta l = K\pi$ with K any integer.

The usual microwave filter design technique[2] for symmetrical (on a β or $1/\lambda$ plot) quarter-wave coupled band-pass filters proceeds as follows. A low frequency lumped constant prototype which has the appropriate frequency characteristic is designed and consists of a cascade of series and shunt resonant circuits all resonant at the same frequency. The frequency characteristics of this prototype are then translated into microwave circuitry[2] which consists of a series of cavities, all resonant at the same frequency but with generally different bandwidths, coupled together with "quarter-wavelengths" of transmission line.

Since θ is independent of frequency, $|R_1|$, and $|R_2|$, it can be seen from (4) that the employment of identical constant-phase two-ports allows one to build the individual resonators of the above structure with fixed resonant frequency, variable bandwidth, and constant "excess phase."[2] The "excess phase" $\pi-\theta$ of all cavities in the structure will furthermore be equal, provided that θ is the same for all the two-ports. This "excess phase" can then be absorbed into the coupling lines which will then remain "quarter-wave" coupling lines at the center frequency for all bandwidths of the individual cavities. The extra selectivity due to the departure of these coupling lengths from quarter wavelengths away from the center frequency is, as usual, counterbalanced by properly selecting the bandwidths of the individual cavities.[2]

Thus any filter whose low frequency prototype consists of a cascade of series and shunt resonators tuned to the same resonant frequency can be synthesized from a cascade of resonant cavities, all of the identical length l and separated by "quarter-wavelength" transformers all of equal lengths, provided that constant-phase two-ports all with the same θ are employed as the discontinuities for the individual cavities. For minimum length cavities and coupling sections,

$$l = \frac{\theta}{\beta_0} \quad \text{and} \quad s = \left(\theta - \frac{\pi}{2}\right)\frac{1}{\beta_0} \tag{5}$$

where β_0 is the propagation constant at the center frequency.

In general, in order to convert a quarter-wave coupled filter into a direct coupled filter the coupling lines of length s and their adjacent discontinuities are combined into single discontinuities with the same reflection coefficients and, excepting a possible change in sign, the same transmission coefficients. We employ (4) to calculate that equivalent reflection coefficient by requiring $\theta - \beta l = \pi/2$ and obtain

$$R_{\text{total}} = \frac{|R_1| + |R_2|}{1 + |R_1 R_2|} e^{i\theta} \tag{6}$$

Thus we may replace this unit with a single constant-phase two-port with the same θ but with the magnitude given in (6). Thus the filter response obtained by the "quarter-wave" coupled configuration can be duplicated by the direct coupled configuration using a smaller number of constant-phase two-ports separated by equal lengths $l = \theta/\beta_0$. The direct coupled configuration is, of course, usually the preferable one due to an over-all compactness and the fact that other passbands are further removed in frequency than for the quarter- (or, as is more common, three-quarter-) wave coupled filter. A similar treatment can be developed for "stagger" tuned structures. The advantages of adjustability are not, in general, as great in such structures and the electrical spacing of the constant phase two-ports varies along the filter.

Experiment

It has been shown[4] that the inclusion of a thin, centered, circular iris, whose diameter is equal to the height of the waveguide, at a variable step-twist junction (see Fig. 2) provides a constant-phase two-port with the following restrictions. 1) $|R|$ can be varied not from zero to unity, but from a minimum value determined by the magnitude of the reflection coefficient of the circular iris alone, when the two sections of waveguide are aligned. 2) $|R|$ and θ are not completely frequency independent. 3) θ becomes a slight function of $|R|$ at the low end of the nominal waveguide band.

Restriction 1) limits the maximum bandwidth attainable with a filter employing these two-ports at a given frequency. Restriction 2) destroys the symmetry of the filter response (on a β plot) for wideband filters but, in practice, this is of little consequence due to the limitation in bandwidth due to one. 2) also causes a decrease in obtainable bandwidth as one goes lower in frequency and an increase as one goes higher in frequency in a given waveguide due to the variation of $|R|$. Restriction 3)

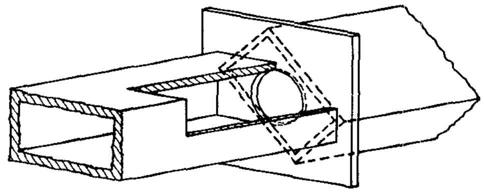

Fig. 2—Modified step-twist junction.

causes a detuning of the filter which is very troublesome and it is likely that the use of oversize waveguide may be necessary for work near the lower end of the nominal waveguide band. This will also, of course, increase the maximum attainable bandwidth at these frequencies.

The transmission characteristics of a symmetrical single cavity step-twist junction plus circular iris filter (see Fig. 3) are presented in Fig. 4. Note that this has the property of variable bandwidth, but constant resonant frequency, previously reported.[4]

In order to test the multiple discontinuity theory a three-cavity direct-coupled filter was constructed as follows. (See Fig. 5.) The center cavity had interior dimensions of 0.520 inch×0.400 inch×0.900 inch with 0.015 inch thick centered circular irises attached to both ends. The two end cavities were formed by attaching centered circular 0.015 inch thick irises to one end of 0.520 inch ×0.400 inch×0.900 inch sections. These were then arranged (Fig. 5) so as to place one iris at each step-twist junction. A cylindrical holder was provided with appropriate slots to allow external rotation of the cavities after assembly and also to allow access to tuning screws, if fine tuning or stagger tuning be desired. Provisions were made for spring loading to keep the cavities in intimate contact while adjustments were made and also for a positive lock once a desired characteristic was obtained. It should be noted that the electrical length of the center cavity can be independently adjusted for the desired center frequency by removing it from the multiple-cavity filter, placing it between aligned waveguides, and inserting the tuning screw until the center frequency of the band-pass characteristic so obtained (see Fig. 4) is the same as that desired for the multiple-cavity filter. The end cavities can also be adjusted in this manner, provided a removable iris is clamped to the end of the cavity not already having one attached.

Having adjusted the electrical lengths of our individual cavities, a tremendous variety of transmission characteristics can now be obtained with our three-cavity filter. We have selected two sets which are appropriate to illustrate its general characteristics. The curves of Fig. 6 were obtained by keeping the two end cavities aligned, *i.e.*, $\delta_1 = \delta_3 = 0$ in Fig. 7, and rotating the center cavity. This arrangement places the lowest values of $|R|$ that we can obtain for this filter at the first and fourth discontinuity. The two inner discontinuities can then be varied from this value of $|R|$ to $|R| = 1$, as the center section is rotated. As can be seen

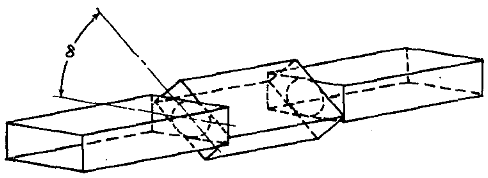

Fig. 3—Single-cavity band-pass filter representation employing modified step-twist junctions as the reflecting elements.

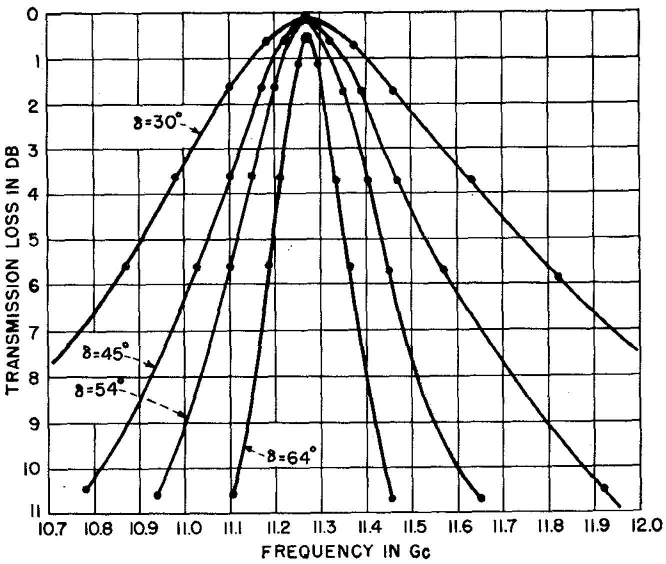

Fig. 4—Transmission characteristics of the filter shown in Fig. 3.

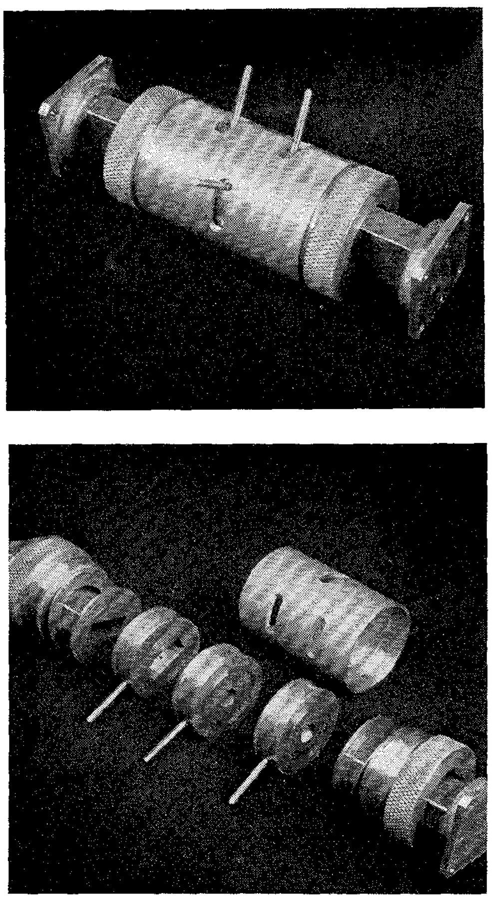

Fig. 5—Photograph of a 3-cavity, direct-coupled, band-pass filter assembly.

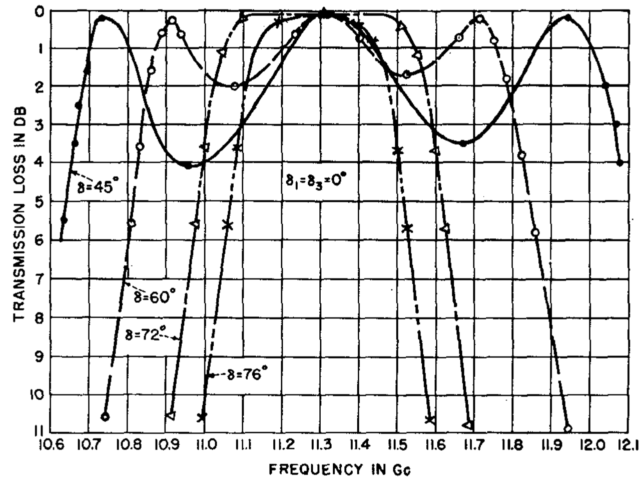

Fig. 6—Transmission characteristics of the filter shown in Fig. 5.

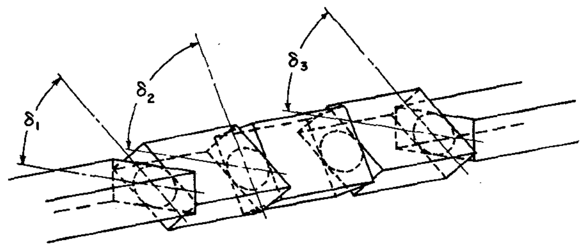

Fig. 7—Three-cavity representation of the filter of Fig. 5 defining angles of rotation.

from Fig. 6, a wide variety of transmission characteristics, including maximally flat, can be so synthesized.

The curves of Fig. 8 were obtained by keeping the two end cavities set at 42° of rotation, *i.e.*, $\delta_1 = \delta_3 = 42°$ in Fig. 7, and rotating the center section. This configuration places higher values of $|R|$ at the 1, 4 positions and thus it is noted that the curve ($\delta_2 = 142°$) approximately corresponding to a maximally flat characteristic is much narrower than the corresponding curve in Fig. 6 ($\delta_2 = 75°$).

The resonant frequency is observed to remain fixed for all these variations in agreement with theory. If still narrower bandwidths are desired for say the maximally flat curve or the 1-db ripple bandpass curve, etc., the two end cavities are simply rotated through larger angles until the desired response is obtained. Although no tuning screws were employed to obtain the results of Figs. 6 and 8, it might be expected that, due to mechanical dissimilarities and losses, some such fine tuning would be necessary for very narrow bandwidths. The tuning screws provided can, of course, shift the characteristics of Figs. 6 and 8 to lower frequencies, should such operation be desired.

By employing an odd number of cavities and a symmetrical structure, the external connecting sections of waveguide are always aligned. This prescription also *always* produces a passband at the frequencies given by $\theta - \beta l = K\pi$. While a rotation of the two external connecting sections of wave guide with respect to one another may be acceptable in certain applications, the

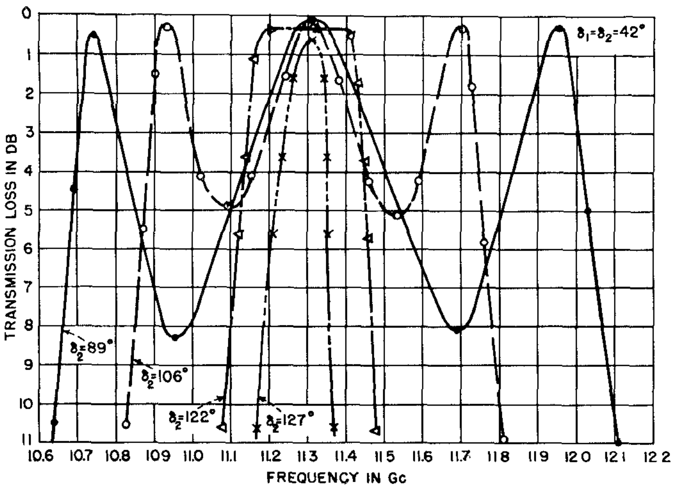

Fig. 8—Transmission characteristics of the filter shown in Fig 5.

advantages for general use of the symmetric, odd cavity configuration with connecting sections aligned are obvious.

Our procedure for obtaining a given characteristic from a direct-coupled cascade is as follows. First, the desired degree of discrimination against frequencies not in the passband, or other pertinent considerations, are used to select the number of cavities desired. An ordinary circular iris cavity, with centered holes whose diameter is the same as the height of the rectangular waveguide, is designed to be resonant at the desired frequency of operation. The iris spacing so obtained is then maintained for all the cavities of the multiple-cavity filter. A cylindrical holder is provided to contain the cavities and allow rotation (see Fig. 5) and, when assembled, the swept transmission and reflection characteristics are simultaneously monitored on a dual-beam oscilloscope. Frequency markers and attenuation calibrations can then be utilized to synthesize the desired filter response.

Conclusions

A theoretical study of the properties of constant-phase two-ports was presented which leads to a simple synthesis technique for a broad class of microwave filters. Constant-phase two-ports consisting of modified step-twist junctions were employed in direct coupled filter configurations whose behavior was predicted from the theory. These filters were shown to have the unique property of adjustable band-pass characteristic and fixed center frequency.

Acknowledgment

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Some Aspects of the Design of Wide-Band Up-Converters and Nondegenerate Parametric Amplifiers*

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Summary—Proper design of the diode-resonating circuit is seen to be extremely important if large bandwidth is desired in a varactor-diode parametric amplifier. Cases where there is one resonance of the diode-resonating circuit at a frequency between the frequencies of the signal-input and the sideband resonances are examined in some detail. It is shown that the frequency of this intermediate resonance can greatly influence the bandwidth capabilities of an amplifier design, and the optimum frequency for such a resonance is given for upper-sideband up-converters. The optimum frequency of such a resonance is greatly different if the diode is resonated in series than it is if the diode is resonated in shunt. It is believed that the same results would also apply for lower-sideband up-converters and nondegenerate parametric amplifiers. Some upper-sideband up-converter designs were worked out and their computed responses are given including the effects of all of the parasitic elements of the diode. Bandwidths of the order of an octave are obtained. A systematic design procedure is given for wide-band nondegenerate parametric amplifiers which use the diode parasitic resistance as the idler termination. Some designs of this type were also worked out and their computed responses (including effects of all diode parasitic parameters) are presented. Bandwidths as large as 33 per cent are obtained depending on the peak gain and operating frequency range.

I. Introduction

It HAS BEEN shown previously[1] that single-diode parametric amplifiers and up-converters using multiple-resonator filters as coupling networks can be made to have considerably larger bandwidths than corresponding amplifiers having single-resonator coupling circuits. The present paper investigates the practical design of the circuitry used to resonate the diode at two

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[1] G. L. Matthaei, "A study of the optimum design of wide-band parametric amplifiers and up-converters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 23–38; January, 1961.